EGMPR+33

Keith Ellis
Fermilab

My work with Graham

- A east coast-west coast (bipolar?) collaboration
- Born of the conviction that the singularities of Feynman diagrams are determined by the Landau rules, and hence come from the propagation of real physical particles.
- Contributed in a significant way to establishing factorization as a property valid beyond leading logarithmic accuracy.
- Influenced the research of others, eg calculation of the NLO splitting kernels
- Factorization is an important issue that underpins all the predictions we make for hard processes at the LHC.

PERTURBATION THEORY AND THE PARTON MODEL IN QCD

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Universal acclaim?

9.11 Critique of conventional treatments

Compared with our presentation so far, a very different approach to factorization is found in much of the literature (e.g., Dissertori, Knowles, and Schmelling, 2003; Ellis, Stirling, and Webber, 1996). It involves a strong emphasis on the mass divergences in massless on-shell partonic reactions, and it asserts that factorization is a method of absorbing mass divergences into a redefinition of parton densities. In contrast, in our presentation the divergences were canceled by subtraction terms that were needed to avoid double counting between, for example, NLO contributions to hard-scattering coefficients and LO contributions.

In this section, we assess the other approach and see that it is physically misleading, if not actually wrong. As such, it is a profound obstacle to further progress in applying perturbative methods to more complicated situations in QCD. Luckily from a practical point of view, the two approaches give the same results for hard-scattering coefficients when parton masses are set to zero. Thus the physical errors do not propagate to numerical results in phenomenology, at least for the simplest reactions.

The approach can be traced back to certain of the early literature on factorization, notably Ellis *et al.* (1979) and Curci, Furmanski, and Petronzio (1980), and it can be summarized as follows:

ellipsis

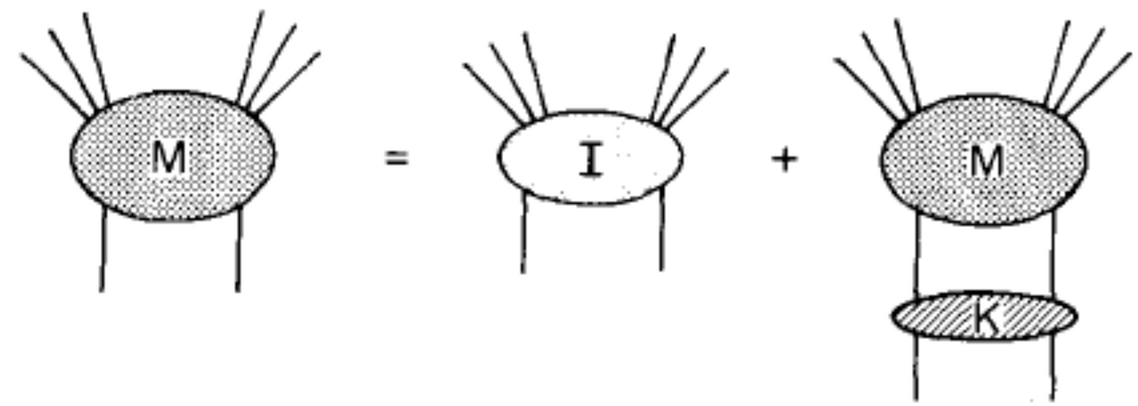
We conclude that it is entirely unphysical to describe the basis of factorization in terms of moving collinear divergences from partonic structure functions or cross sections into redefined parton densities. Naturally, attempting to extend an incorrect method to more general situations leads to a conceptual morass. It is more by luck than good physics that the same hard-scattering coefficients are obtained for standard reactions.

J.C. Collins, Foundations of perturbative QCD
Cambridge University Press (2011)

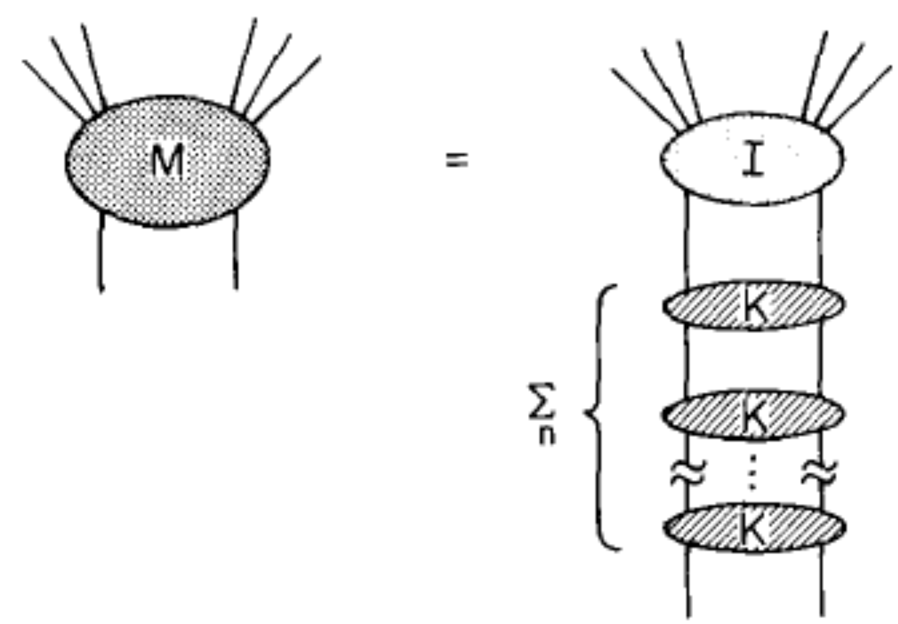
Generalized ladder diagrams

- Division of cut diagrams into 2PR (two particle reducible) and 2PI (two particle irreducible)
- Examination of the kinematics of ladder diagrams
- A series of projections on to physical states preserving the 2PI nature
- The 2PI kernels are functions of the input and output momenta K(k′,k)

The generalized ladder equation



...and its formal solution



Inserting $\mathcal{P}$ (a projector onto physical states) we have,

$$\begin{aligned}
\mathcal{M} &= I\Big[\frac{1}{1-K}\Big] \\
&\equiv \Big[I\frac{1}{(1-(1-\mathcal{P})K)}\Big]\frac{(1-(1-\mathcal{P})K)}{(1-K)} \\
&\equiv \Big[I\frac{1}{(1-(1-\mathcal{P})K)}\Big]\frac{(1-(1-\mathcal{P})K)}{(1-(1-\mathcal{P})K-\mathcal{P}K)} \\
&\equiv \Big[I\frac{1}{(1-(1-\mathcal{P})K)}\Big]\frac{1}{(1-\mathcal{P}K')}, \quad K' = K\frac{1}{1-(1-\mathcal{P})K}.
\end{aligned}$$

The formal denominators are defined via the series expansion

$$\frac{1}{1-\mathcal{P}K'} = 1 + (\mathcal{P}K') + (\mathcal{P}K')(\mathcal{P}K') + \ldots$$

Kinematics of cut ladder diagrams

From branching kinematics $k_T^2 \leq -k^2(1-\beta)$ so we may define $k_T = \sqrt{-k^2}\kappa$

$$
\begin{aligned}
k &= \beta p - \alpha n + k_T \\
\alpha &= \frac{\beta^2 p^2 - k^2 + k_T^2}{2\beta n \cdot p} \equiv \frac{\beta^2 p^2 - k^2(1-\kappa^2)}{2\beta n \cdot p}
\end{aligned}
$$

From the condition that $p - k$ is timelike and has positive energy we have that

$$\beta > 0, \quad \alpha > 0$$

Thus for $\beta > 0$, the conditions $p^2, k^2 \to 0$ imply $k = \beta p$.

The kinematics of cut ladder diagrams allow us to reduce the full four momentum integration to a convolution, ($d^4k \to d\,\beta$)

Oversubtraction

Defining $\tilde{I}(k) = I(k)|_{k^2=p^2=0} = \tilde{I}(\beta p)$ and $\tilde{K}(k', k)|_{k^2=p^2=0} = K\tilde{(}k, \beta p)$.

$$\Delta I = I - \tilde{I}, \;\; \Delta K = K - \tilde{K}$$

$$\begin{aligned} \mathcal{M} &= I \frac{1}{1-K} \equiv \tilde{\mathcal{M}} \frac{1}{1-K} + \Delta I \frac{1}{1-\Delta K} \\ \tilde{\mathcal{M}} &= \tilde{I} + \Delta I \frac{1}{1-\Delta K} \tilde{K} \end{aligned}$$

In the second term every term is oversubtracted and of order p^2.

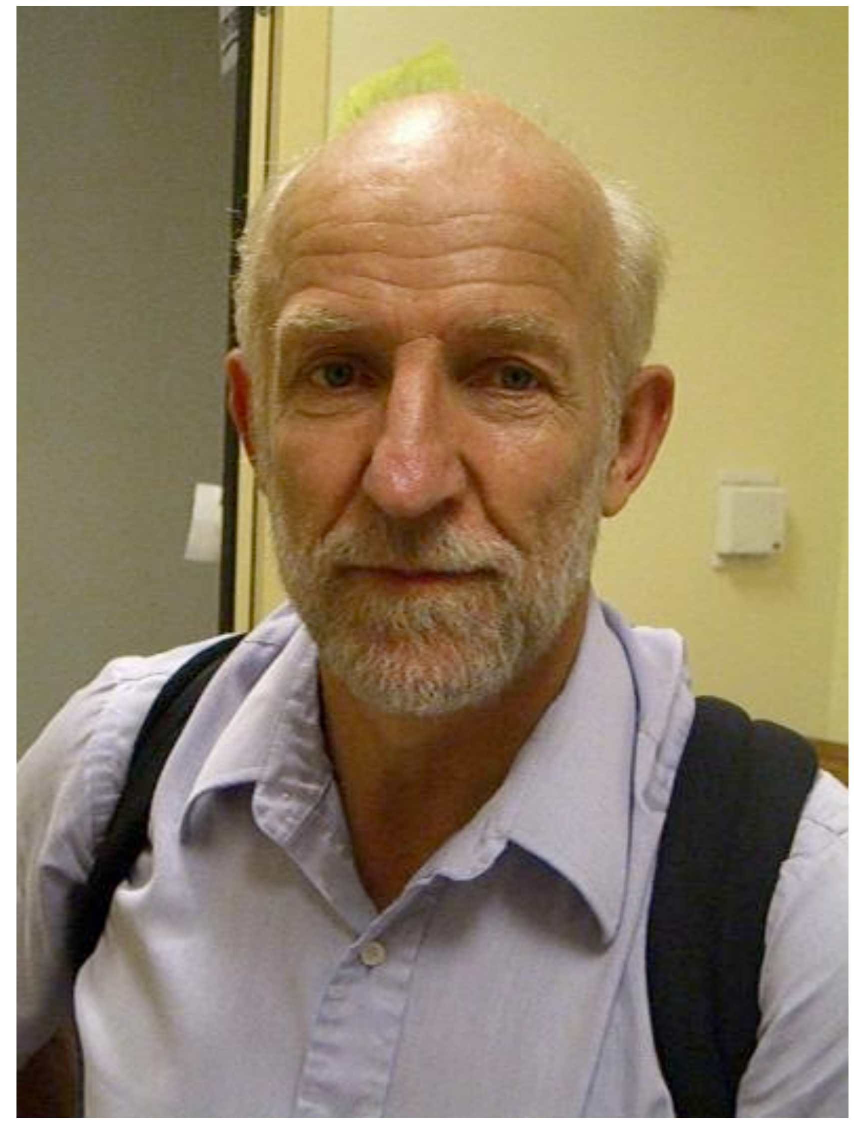
$$\Delta I \frac{1}{1-\Delta K} = \sum_{n=0}^{\infty} \Delta I (\Delta K)^n$$

We end up with a parton model type formula and an explicit expression for the anomalous dimension function.

$$\begin{aligned} \mathcal{M}(p) &= \int_0^1 \tilde{\mathcal{M}}(\beta p)\, \gamma(\beta) \frac{d\beta}{\beta} + O(p^2) \\ \gamma(\beta) &= \beta \int d^4k\, \delta\left(\beta - \frac{n \cdot k}{n \cdot p}\right) [\frac{1}{1-K}(k, p)] \end{aligned}$$

The heavy lifting

- The real challenge is to demonstrate that the 2PI kernels are free from singularities so that the limits $k^2=0$ can be taken.
- In this we were greatly helped (at least in the collinear region) by the adoption of a physical gauge, in which the unphysical degrees of freedom do not propagate.
- This gives extra factors in the numerator, as can be understood from a helicity argument.
- The detailed arguments are based on power counting.
- Treatment of soft and collinear regions separately.



Missing region: Glauber scattering

- Glauber region defined as $k^+ k^- < k_T^2$
- In the Glauber region $(p_A + k)^2 - m^2 = k^2 + 2p_A \cdot k \neq 2p_A \cdot k.$
- Proofs of factorization rely on the use of the Ward identity.

$$A_\mu g^{\mu\nu} B_\nu \approx A^+ B^- \approx \frac{(A \cdot k)(k \cdot B)}{k^+ k^-}, \quad k^\pm = k^0 \pm k^3.$$

- $A^+ k^- \to A \cdot k$ valid only if all the components of k are roughly the same.
- Often (but not for DY), one can deform the counter out of the Glauber region.
- At least for DY processes the factorization has been established.
- In the centre of mass system the Glauber singularities cancel because of a combination of the colour singlet nature of B, unitarity, and the time dilation of the interactions of the constituents of B.

Bodwin, Brodsky, Lepage, PRL 47 (1981) 1799
Lindsay, D.A. Ross and Sachrajda, NPB214 (1983) 61
Collins, Soper, Sterman, hep-ph/0409313

The follow-up paper: Curci, Furmanski and Petronzio

Nucl.Phys.B175:27,1980.

* Followed exactly the procedure laid out in EGMPR.
* Introduced specific forms for the projectors.
* Provided compact forms for the two loop anomalous dimensions, eg the qq splitting function.
* Identified small errors in the P_{gg} as calculated (previously) using the operator product expansion.

$$
\begin{aligned}
P_{qq}^{(1)} \quad = \quad & C_F^2\left\{-1+x+\left(\frac{1}{2}-\frac{3}{2}x\right)\ln x-\frac{1}{2}(1+x)\ln^2 x\right.\\
&\left.-\left[\frac{3}{2}\ln x+2\ln x\ln(1-x)\right]p_{qq}(x)+2p_{qq}(-x)S_2(x)\right\}\\
&+C_F C_A\left\{\frac{14}{3}(1-x)+\left[\frac{11}{6}\ln x+\frac{1}{2}\ln^2 x+\frac{67}{18}-\frac{\pi^2}{6}\right]p_{qq}(x)\right.\\
&\left.-p_{qq}(-x)S_2(x)\right\}\\
&+C_F T_f\left\{-\frac{16}{3}+\frac{40}{3}x+\left(10x+\frac{16}{3}x^2+2\right)\ln x\right.\\
&\left.-\frac{112}{9}x^2+\frac{40}{9x}-2(1+x)\ln^2 x-\left[\frac{10}{9}+\frac{2}{3}\ln x\right]p_{qq}(x)\right\}
\end{aligned}
$$

Aberdonians

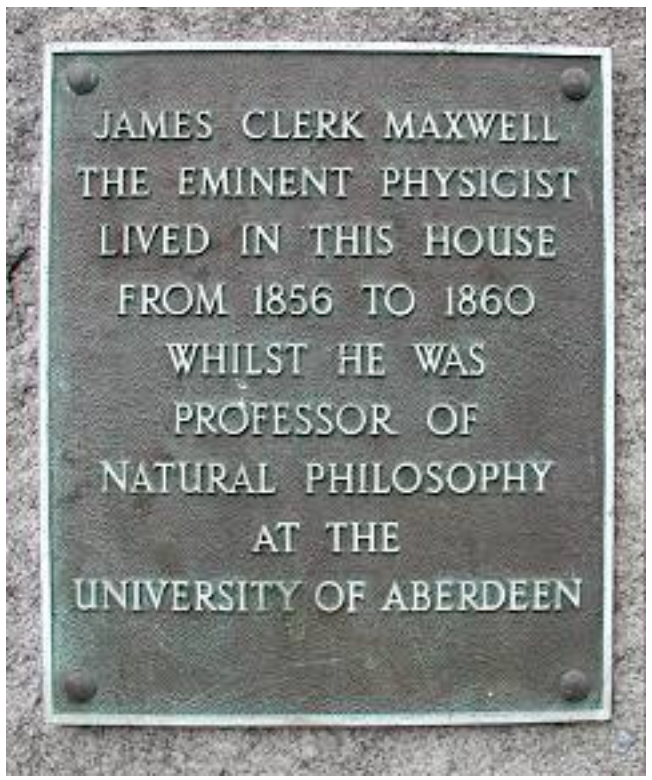
How others see us...

and how we see ourselves......

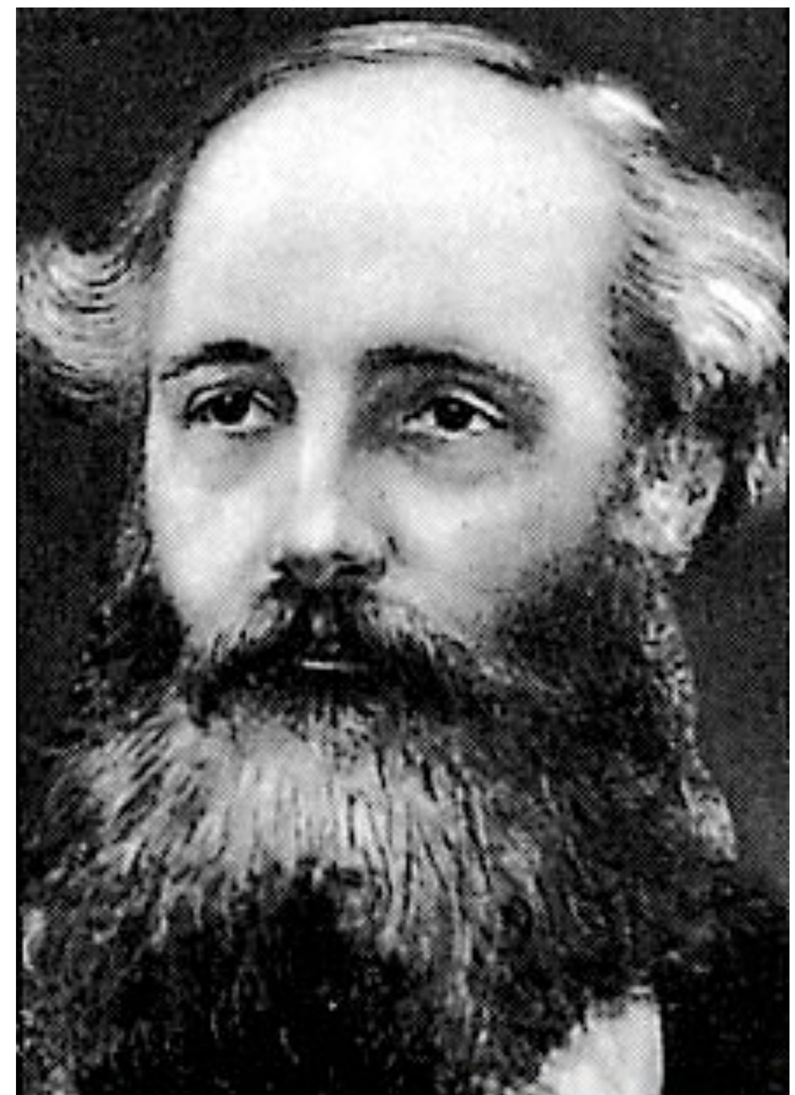
...generous to a fault, happy and hirsute

STOP

The greatest physicist who spent his young life in Aberdeen

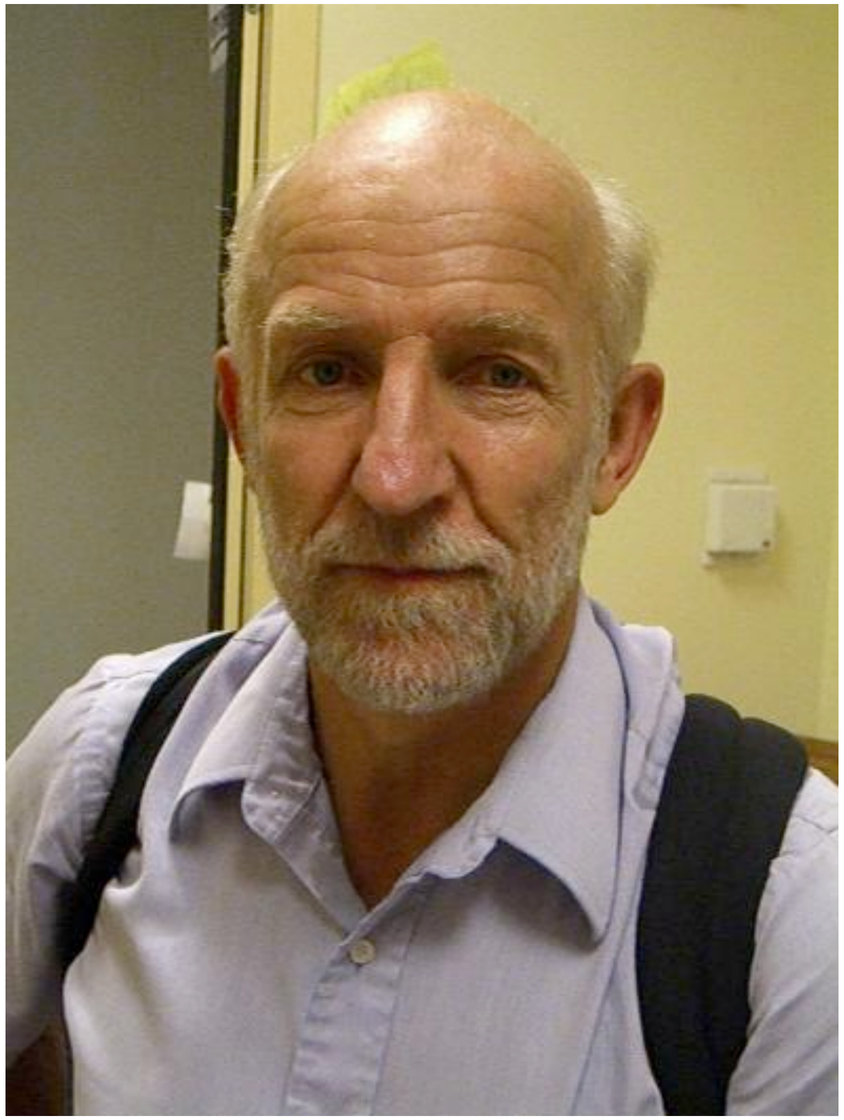


131 Union Street, Back Wynd Steps



Maxwell and Aberdeen

- Aberdeen had two universities King’s College, established in 1495 by papal bull and Marischal College established 1593, a protestant institution, for the training of post-reformation clergy.
- Maxwell was professor in Aberdeen at Marischal College (1856-1860)
- Worked on the stability of Saturn’s rings.

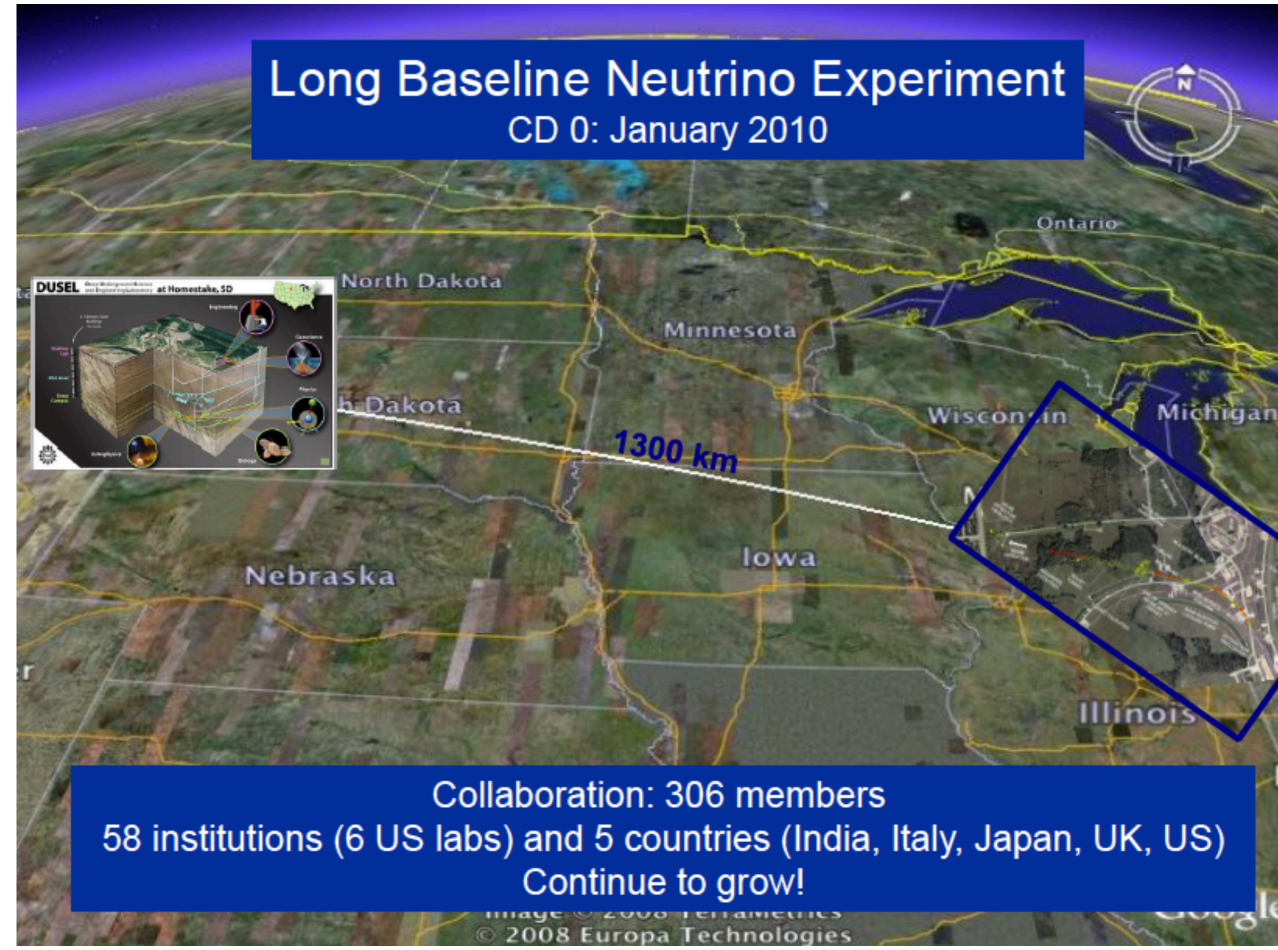


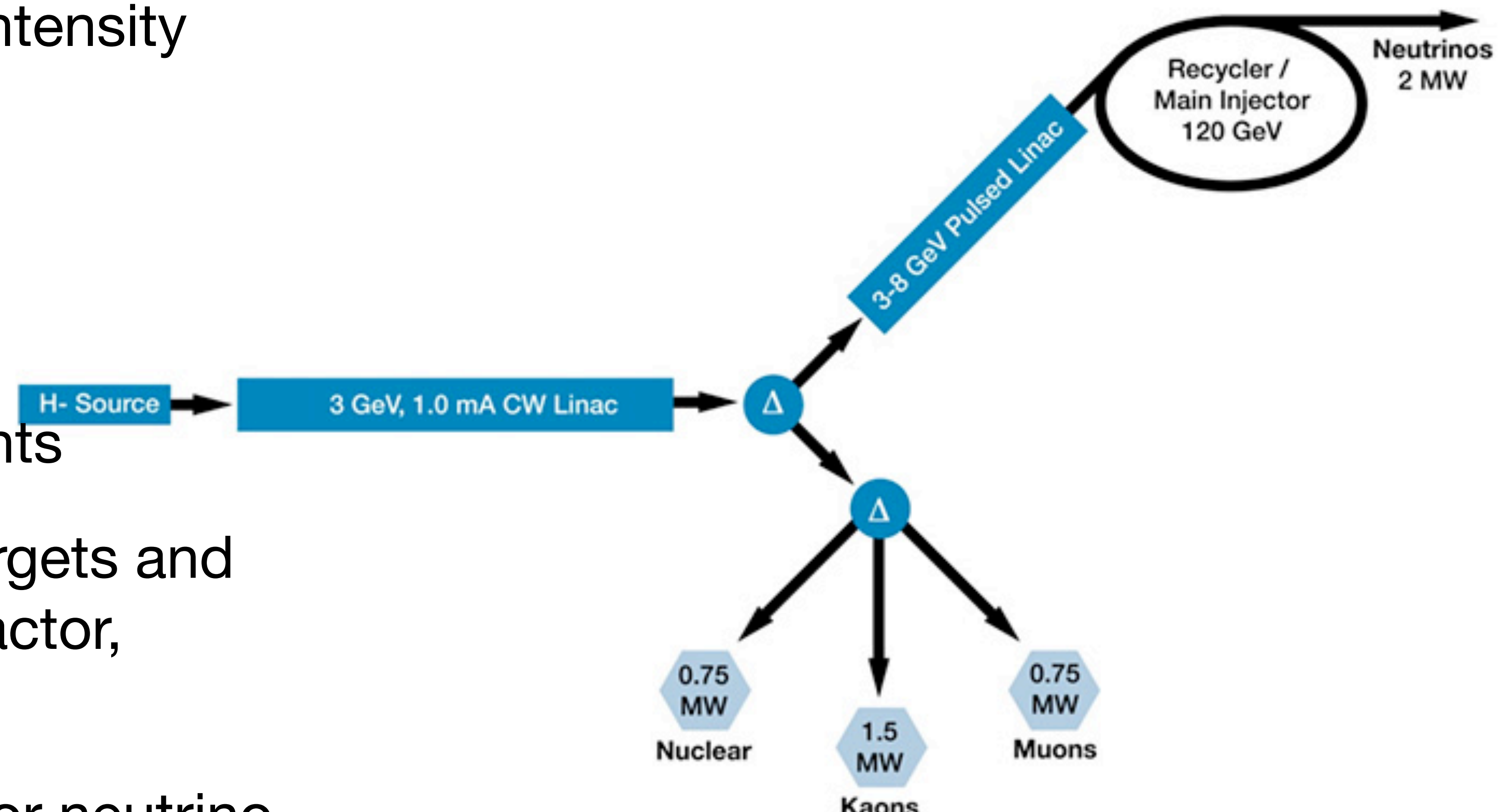
Graham,

Lang may yer lum reek!

Program for the next decade

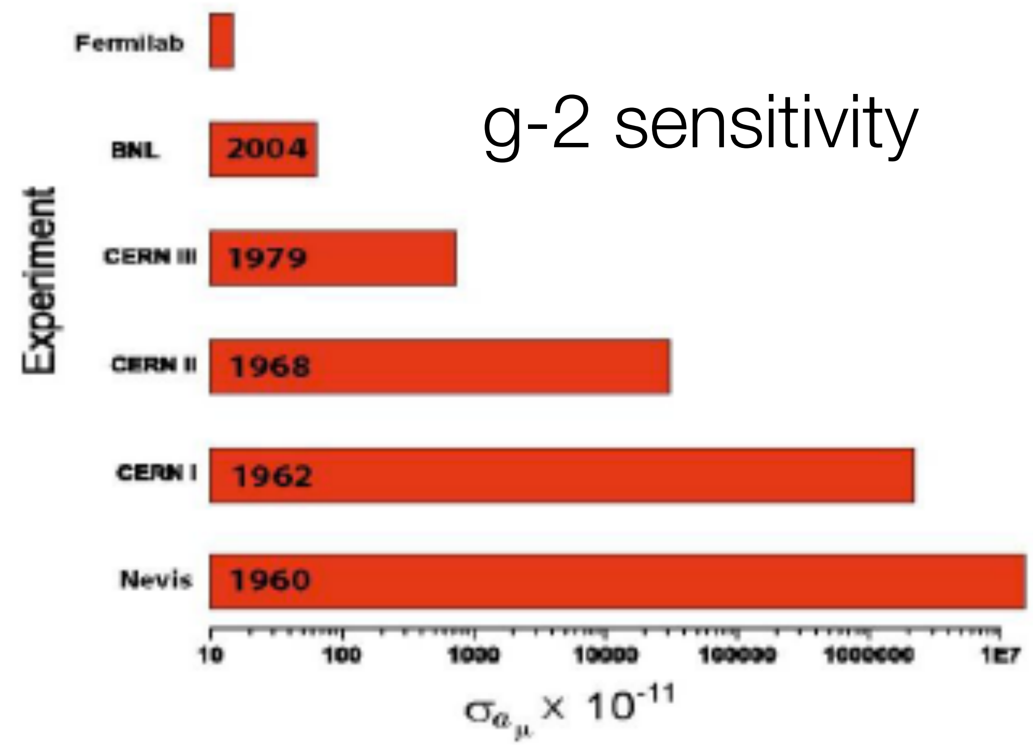
- LBNE - Long baseline neutrino experiment (baseline 1200km) to South Dakota
  - Neutrino mass spectrum (mass hierarchy)
  - Matter-Antimatter symmetry
  - Neutrino anti-neutrino differences
- Project X - Megawatts of continuous beam a world-leading facility for the intensity frontier
  - >2MW to LBNE
  - Kaon experiments
  - Rare muon decay experiments
  - Applications to spallation targets and ADS (sub-critical nuclear reactor, accelerator driven)
  - Front end for muon collider or neutrino factory.





Intensity Frontier: Rare processes

- g-2: anomalous magnetic moment of the muon x20 statistics



- Mu2e: direct muon to electron conversion - huge sensitivity to NP:Single event sensitivity below 10^{-16}

- SeaQuest: nuclear physics Drell-Yan process to study the structure of the nucleon in the nuclear environment.

Intensity Frontier:Neutrinos

- ν Standard model: Pattern of neutrino masses and mixings.
  - MINOS, Nova, LBNE
- ν beyond the standard model: the search for sterile neutrinos and anomalous interactions.
  - Short baseline: MiniBoone-MicroBoone
  - Long-baseline: MINOS, Nova
- Neutrino physics measurements as a probe of nuclear structure and support of oscillation experiments
  - Dedicated experiment: Minerva

Nova building and 15KTon detector schematic